Two-Dimensional DOA Estimation Based on L-shaped Array

Huang Liwei, Chen Yulin, Huang Hui

Southeast University Chengxian College, State Key Laboratory of Millimeter Waves Nanjing China

Keywords: Direction of Arrival (DOA), L-shaped array, Root-MUSIC, ESPRIT, MEMP

Abstract: Based on L-shaped array, three two-dimensional direction-of-arrival (2-D DOA) estimation algorithms are investigated. Root-MUSIC algorithm for 2-D DOA estimation obtains parameter via getting the roots of a polynomial. ESPRIT algorithm for 2-D DOA estimation uses the rotational invariance principle of matrix without spectral peak searching. Matrix Enhancement and Matrix Pencil(MEMP) algorithm forms a new augmented matrix by using covariance matrix of the receiving source signals. The three algorithms for L-shaped array are studied and simulated with MATLAB. The performance shows that high resolution, estimation precision and stability can be obtained with different SNRs.

1. Introduction

Direction-of-arrival (DOA) estimation is a basic problem of array signal processing. L-shaped array, which has simple structure and is easy to be implemented, is often used in communication areas. Azimuth angle and elevation angle are estimated in two-dimensional direction-of-arrival (2-D DOA) estimation for L-shaped array. This paper compares the characteristics of Root-MUSIC, ESPRIT and Matrix Enhancement and Matrix Pencil(MEMP) algorithms for 2-D DOA estimation with L-shaped array and simulates in different conditions with MATLAB.

2. Model of L-shaped array

Compared to other two-dimensional arrays, the structure of L-shaped array is simply. With less numbers of receiving array elements and high estimation precision, L-shaped array is used widely. Figure(1) is one structure of its model[1].

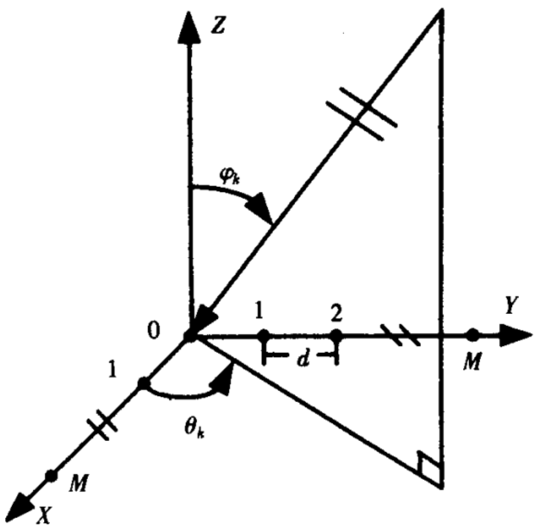

Fig. 1 Model structure of L-shaped array

Suppose there are M array elements with uniform distribution in x-axis and y-axis respectively. The distance between adjacent array elements is d. There are K source signals with wavelength of λ_K and angle (θ_K φ_K) in the space, and θ_K and φ_K are azimuth angle and elevation angle of the signal.

The L-shaped array can be regarded as the superposition of two linear arrays. So the direction

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vectors of the x-axis and y-axis are similar to the linear array. They can be defined as

$$\alpha_x(\theta_k,\varphi_k)=[1,u(\theta_k,\varphi_k),\ldots,u^N(\theta_k,\varphi_k)]^T \tag{1}$$

$$\alpha_y(\theta_k,\varphi_k)=[v(\theta_k,\varphi_k),v^2(\theta_k,\varphi_k)\ldots,v^N(\theta_k,\varphi_k)]^T \tag{2}$$

$u(\theta_k,\varphi_k)$ and $v(\theta_k,\varphi_k)$ are expressed as formulas(3) and (4)

$$u(\theta_k,\varphi_k)=\exp[j2\pi d\cos\theta_k\sin\varphi_k/\lambda] \tag{3}$$

$$v(\theta_k,\varphi_k)=\exp[j2\pi d\sin\theta_k\sin\varphi_k/\lambda] \tag{4}$$

So the direction vectors of the received signal of x-axis and y-axis can be expressed as

$$A_x=[\alpha_x(\theta_1,\varphi_1),\alpha_x(\theta_2,\varphi_2),\ldots,\alpha_x(\theta_k,\varphi_k)] \tag{5}$$

$$A_y=[\alpha_y(\theta_1,\varphi_1),\alpha_y(\theta_2,\varphi_2),\ldots,\alpha_y(\theta_k,\varphi_k)] \tag{6}$$

Suppose S is the matrix of source signals, N_x and N_y are noise signals received by x-axis and y-axis respectively. The vector representations of signals received by x-axis and y-axis are represented as

$$X=A_xS+N_x \tag{7}$$

$$Y=A_yS+N_y \tag{8}$$

3. Two-dimensional DOA estimation with L-shaped array

3.1 Two-dimensional estimation with L-shaped array based on Root-MUSIC algorithm

Root-MUSIC algorithm obtanis parameter estimation via getting the roots of the polynomial. This algorithm does not require peak searching, so the computational complexity of parameter estimation is reduced obviously compared to classical MUSIC algorithm.With the same mean square errors(MSE), the average numbers of angle estimation error with Root-MUSIC algorithm are much less than with classical MUSIC algorithm[2].

When using Root-MUSIC algorithm to get 2-D DOA estimation parameters with L-shaped array, it is needed to obtain the signals received by the receiving array on x-axis and y-axis respectively according to the model structure in figure(1). The L-shaped array is equivalent to the superposition of two uniform linear arrays, so x-axis and y-axis are treated in the same way.

Firstly, the signals obtained on the x-axis are processed, and the received signals are used to construct matrix $R_{XX}=XX^H/N$, where R_{XX} is the covariance matrix. It can be decomposed into signal subspace U_{Xs} and noise subspace U_{Xn}. And defines P(z) by formula(9)

$$P(z)=[1,\ z,\ldots,\ z^{M-1}]^T \tag{9}$$

The peak searching represation is described as formula(10)

$$f(\theta,\phi)=\frac{1}{a_X^H U_{Xn} U_{Xn}^H a_X} \tag{10}$$

Make use of the orthogonality of noise space and direction vector to find zeros of the denominator in formula(10). As to the orthogonality of noise space and direction vectors, it equals to get zeros of the equation as

$$P^H(z)U_{Xn}U_{Xn}^H P(z)=0 \tag{11}$$

Find N roots whose absolute value closest to 1 for the Root-MUSIC algorithm polynomial, the roots are described as

$$f(z) = z^{M-1}P^T(z^{-1})U_{Xn}U_{Xn}^H P(z) \tag{12}$$

And then, after further calculation, the parameter estimation of x-axis can be obtained by formula (13)

$$u=-\mathrm{angle}(z_x)\lambda/2\pi d \tag{13}$$

Since the x-axis and y-axis are treated in the same way, the parameter estimation of the y-axis can be obtained similarly by formula(14)

$$v=-\mathrm{angle}(z_y')\lambda/2\pi d \tag{14}$$

Finally, after some angle pairing calculations, the two-dimensional angle parameters can be estimated according to formula(15) and(16), and u_i and v_j are mateched pairs.

$$\theta_K=\tan^{-1}\left(\frac{v_j}{u_i}\right) \tag{15}$$

$$\varphi_K=\sin^{-1}\left(\sqrt{u_i^2+v_j^2}\right) \tag{16}$$

Simulations of Root-MUSIC algorithm use 3 separate source signals. The matrix of azimuth angle is [15 30 45] and the matrix of elevation angle is [10 30 50]. The number of snapshots is 100.There are 8 array elements on the x-axis and y-axis respectively, with the distance of 0.5 meters between two adjacent arrary elements. Take different signal-to-noise ratios(SNR) for simulation, the results are shown in table 1.

Table 1 Simulation of Root-MUSIC algorithm with L-shaped array under different SNRs

SNR	results			
		angle 1	angle 2	angle 3
10	azimuth angle	16.0817	30.9798	44.3724
	elevation angle	9.9049	29.3133	50.3863
20	azimuth angle	14.7907	30.2262	44.8943
	elevation angle	10.0237	29.9968	50.0397
30	azimuth angle	15.1229	30.0085	44.9899
	elevation angle	9.9658	29.9867	50.0146

Table 1 shows that the perfomence of Root-MUSIC algorithm for 2-D DOA estimation with L-shaped array is good at different signal-to-noise ratios. Generally, with the increase of signal-to-noise ratio, the resolution improves.

3.2 Two-dimensional estimation with L-shaped array based on ESPRIT algorithm

The ESPRIT algorithm for 2-D DOA estimation uses the rotational invariance principle of matrix to decompose the signal matrix to obtain subspaces and get estimation values of azimuth angle and elevation angle. Compared with classical MUSIC algorithm, the 2-D ESPRIT algorithm does not require spectral peak searching, which simplifies the computational complexity. Compared with conventional ESPRIT algorithm, it solves the problem of parameter pairing [1][3].

When using 2-D ESPRIT algorithm to get DOA estimation parameters with L-shaped array, it is needed to obtain the signals received by the receiving array elements on the x-axis and y-axis respectively according to the model structure in figure(1).

Firstly, the direction matrix of the x-axis is divided into two submatrixes. One submatrix is composed of direction vectors from the first matrix element to the N-1 matrix element, and the other is composed of direction vectors from the second matrix element to the N matrix element. Similarly, the direction matrix of the y-axis is divided into two submatrixes. The four submatrixes are named as X1, X2, Y1 and Y2.

Secondly, according to X1,X2,Y1 and Y2, three cross correlation matrixes can be made and named as C1, C2 and C3 as formula(17)

$$C_1=R_{X1Y1} \quad C_2=R_{X2Y1} \quad C_3=R_{X2Y2} \tag{17}$$

And then, matrix $C=[C_1,C_2,C_3]^T$ is formed. By singular value decomposition of matrix C, the signal subspace E_S of the received signals can be obtained, which is expressed as

$$E_S = \begin{bmatrix} E_0 \\ E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} A \\ A\Phi_x \\ A\Phi_x\Phi_y^H \end{bmatrix} T \tag{18}$$

A is obtained by direction matrix A_x and A_y of the receiving signals. Ψ_x and Ψ_y are expressed as

$$\Psi_x=(E_0^H E_0)^{-1}E_0^H E_1 \tag{19}$$

$$\Psi_y=(E_1^H E_1)^{-1}E_1^H E_2 \tag{20}$$

Matrix Φ_x and Φ_y^H can be obtained by eigenvalue decomposition of Ψ_x and Ψ_y. It is needed to rearrange the eigenvalue matrix according to the eigenvalue. And pairing algorithm is needed. After pairing, finally, the estimation values can be obtained by formulas(21) and (22)

$$\theta_k=\tan^{-1}\left(\frac{\mathrm{angle}(v(\theta_k,\varphi_k))}{\mathrm{angle}(u(\theta_k,\varphi_k))}\right) \tag{21}$$

$$\varphi_k=\sin^{-1}\left(\frac{\lambda}{2\pi d}\sqrt{v^2(\theta_k,\varphi_k)+u^2(\theta_k,\varphi_k)}\right) \tag{22}$$

Simulations of 2-D ESPRIT algorithm for DOA estimation use 3 separate signal sources.The matrix of azimuth angle is [15 30 45] and the matrix of elevation angle is [10 30 50]. The number of snapshots is 200.There are 8 array elements on the x-axis and y-axis respectively, with the distance of 0.5 meters between adjacent array elements. Take different signal-to-noise ratios(SNR) for simulation, the results are shown in table 2.

Table 2 Simulation of 2-D ESPRIT algorithm with L-shaped array under different SNRs

SNR	results			
		angle 1	angle 2	angle 3
10	azimuth angle	14.7380	29.7591	45.5334
	elevation angle	10.2621	30.0405	49.4228
20	azimuth angle	15.1303	29.8543	45.0168
	elevation angle	10.0277	30.1182	50.0409
30	azimuth angle	15.1772	30.0426	45.0088
	elevation angle	9.9719	29.9383	50.0077

Table 2 shows that the perfomence of ESPRIT algorithm for 2-D DOA estimation with L-shaped array is good at different signal-to-noise ratios. It is of practical utility to use 2-D ESPRIT algorithm for DOA estimation.

3.3 Two-dimensional estimation with L-shaped array based on MEMP algorithm

The essence of MEMP algorithm is to form a new augmented matrix by using the covariance matrix of the receiving source signals[4]. This method does not require a lot of sampling data. The speed of operation is very fast and the precision of operation is very high. Matrix model is needed to collect data before the augmented matrix is defined. Since L-shaped array needs less numbers of array elements compared with other array model, the array utilization ratio of this algorithm is low.[5]

When using MEMP algorithm to get 2-D DOA estimation parameters with L-shaped array, signals received by the receiving array elements on the x-axis and y-axis are described as matrix representations, and cross covariance matrix of x-axis and y-axis received vectors is constructed by formula (23)

$$R_{XY} = A_x S A_y^H + \sigma^2 E \tag{23}$$

Secondly, use the cross covariance matrix to form augmented matrix Re[4]. Decompose Re to get K large eigenvalues, and use the eigenvectors of K eigenvalues to form matrix U_{SX}, which represents the subspace of the signal. Decompose U_{SX} into two matrixes U_{x1} and U_{x2}, define matrix wtih fomula(24)

$$F_x = U_{x1}^{\#} U_{x2} \tag{24}$$

Because U_{x1} is not a square matrix, inverse matrix can not be obtained directly, but generalized inverse matrix can be obtained by $U_{x1}^{\#}$. With the same way, formula(25) can be obtained easliy. C is permutation matrix described as formula (26). [5]

$$F_y = (CU_{x1})^{\#}(CU_{x2}) \tag{25}$$

$$C = \sum_{k=1}^{Q}\sum_{l=1}^{P} C_{k,l}^{Q\times P} \otimes C_{l,k}^{P\times Q} \tag{26}$$

The estimated values of uk and vk can be obtained by eigenvalue decomposition of Fx and Fy. Pairing algorithm is used for uk and vk.

Finally, the estimation values can be obtained by formulas(27)(28)

$$\theta_k = \tan^{-1}\left(\mathrm{angle}(v_k) \big/ \mathrm{angle}(\mu_k)\right) \tag{27}$$

$$\varphi_k = \cos^{-1}\left(\frac{\lambda}{2\pi d}\sqrt{\mathrm{angle}^2(\mu_k) + \mathrm{angle}^2(v_k)}\right) \tag{28}$$

Simulations of MEMP algorithm use 3 separate signal sources.The matrix of azimuth angle is [15 30 45] and the matrix of elevation angle is [10 30 50]. The number of snapshots is 200.There are 8 array elements on the x-axis and 10 array elements on the y-axis. With the distance of 0.5 meters between adjacent arrary elements. Take different signal-to-noise ratios(SNR) for simulation, the results are shown in figure 2,3,4 and 5.

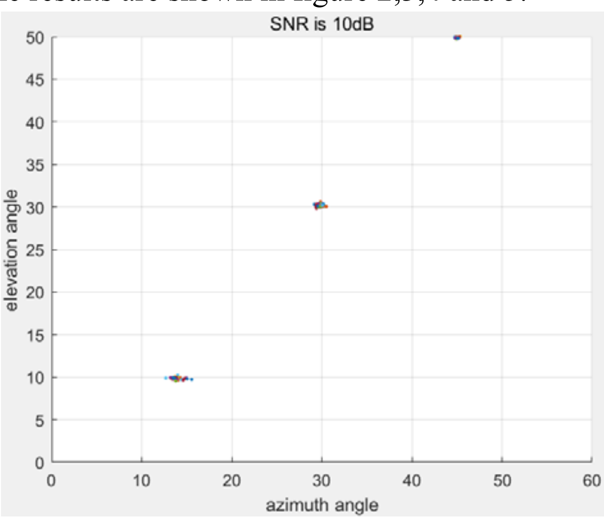

Fig.2 Simulation of MEMP algorithm with SNR 10

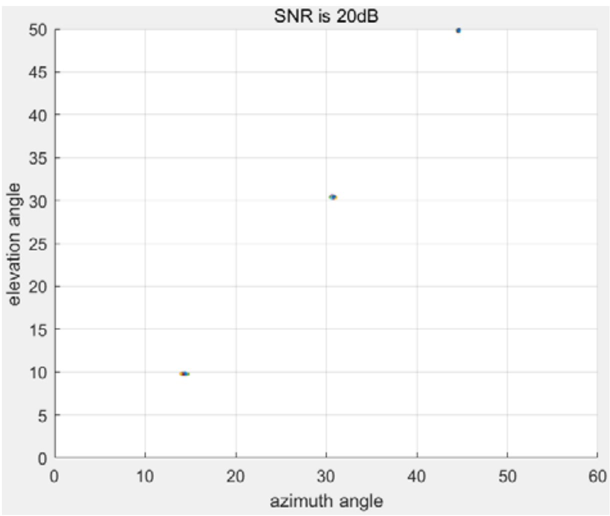

Fig.3 Simulation of MEMP algorithm with SNR 20

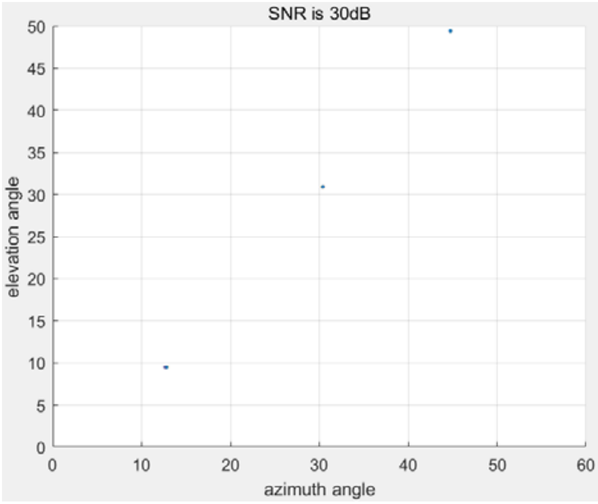

Fig.4 Simulation of MEMP algorithm with SNR 30

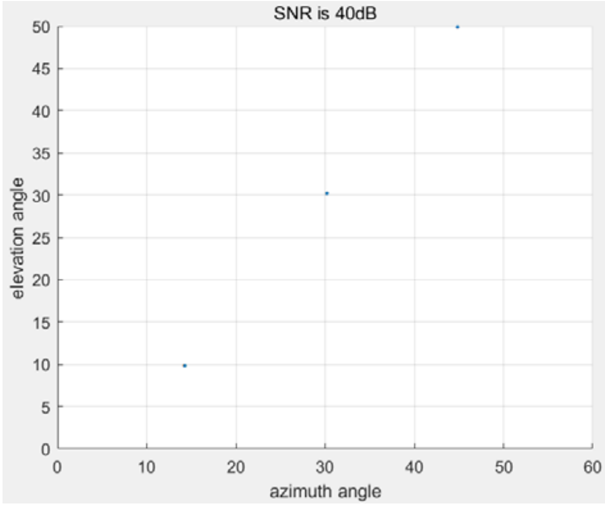

Fig.5 Simulation of MEMP algorithm with SNR 40

Figures 2,3,4 and 5 show the simulation results of 2-D DOA estimation based on MEMP algorithm with different SNRs. From the value of estimation, it is easily to know that this algorithm can estimate DOA of sources with high precision. The algorithm avoids spectrum peak searching, so it has fast calculation speed. But it reduces numbers of effective array elements, which may lead to noise interference.

4. Conclusion

This paper analyses three 2-D DOA estimation algorithms for L-shaped array. The principles of Root-MUSIC, ESPRIT and MEMP algorithms for 2-D DOA estimation are investigaed. The simulation with different SNRs are done respectively. The Root-MUSIC algorithm obtains parameter via getting the roots of a polynomial, the 2-D ESPRIT algorithm uses the rotational invariance principle of matrix and MEMP algorithm constructs a new augmented matrix by using covariance matrix of the receiving source signals. The performance shows that high resolution, estimation precision and stability can be obtained with different SNRs for DOA estimation.

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